An Adaptive Symplectic Integrator to Model the Mechanics of Self-Gravitating Systems

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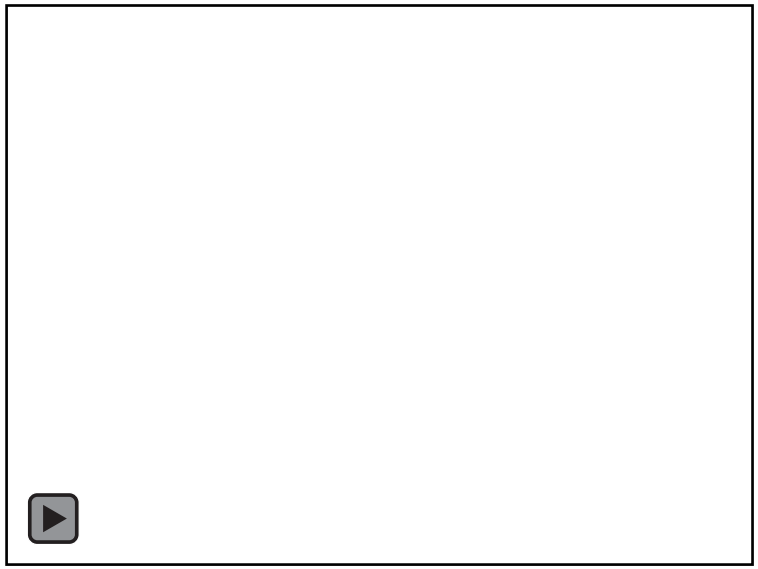
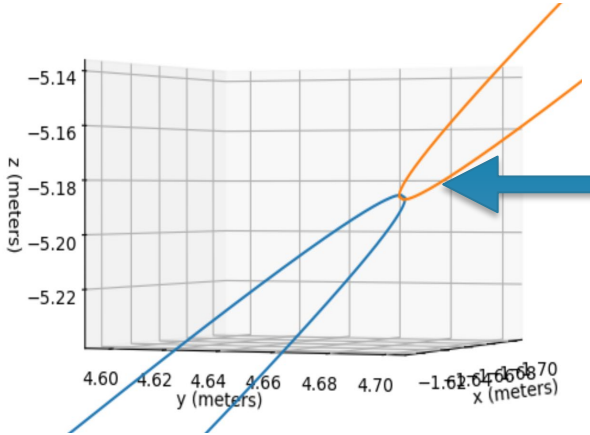


Introduction



Two Particles

[1]

Billions of Particles

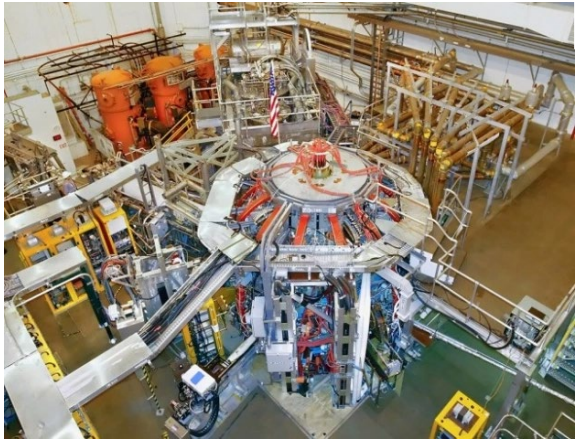
The middle-range (10s, 100s, 1000s)

[1]: *ESA/Hubble & NASA, D. Rosario et al.*

Why Understand/Model These Systems?

- Here at PPPL: Keeping fast, hot ions within range for fusion in tokamaks like NSTX-U

- Interesting Questions
  - Stability of our Solar System
  - Possibility of “Planet 9” being a planet-mass black hole



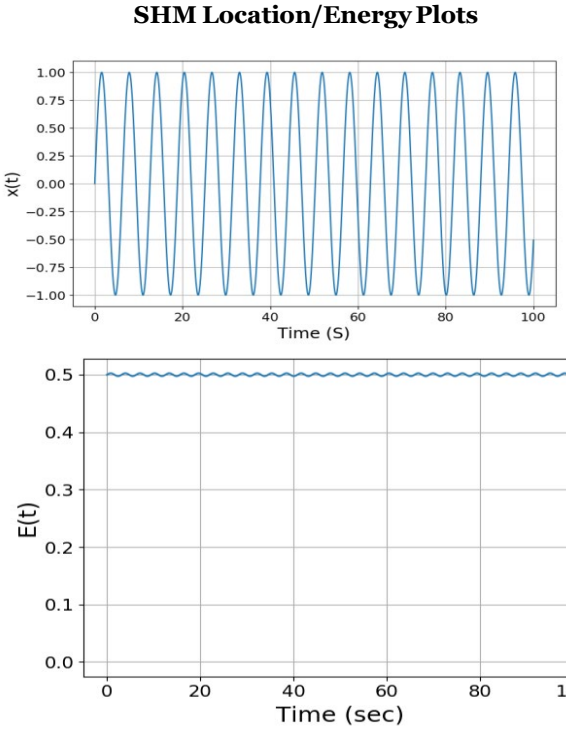
[2]

[3]

[2]: *Outside view of the NSTX reactor [Digital image]. (n.d.). Retrieved from https://en.wikipedia.org/wiki/National_Spherical_Torus_Experiment#/media/File:NSTX.jpg*
[3]: *[Digital image]. (2019, November 22). Retrieved August 08, 2020, from https://www.nasa.gov/ames/press-release/nasa-selects-new-research-teams-to-further-solar-system-exploration-research*

Feasibility Test – Simple Harmonic Oscillator

- Begun with modeling the motion and energy of the simple harmonic oscillator
  - Modeled under a second-order leapfrog scheme
  - Computed and plotted in Python
- Accurate “Location Plot”
- Having no outside forces should yield a constant total energy plot, but it’s sinusoidal instead- characteristic of all leapfrog integrators.

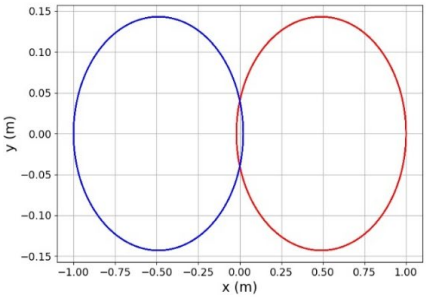


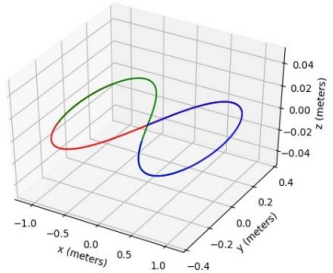
The Two/Three Body Problem

- Plots of the two and three body problem under the leapfrog scheme with given test parameters
  - Computed with C, plotted with Python
  - Computation was done between each pair of particles in the system

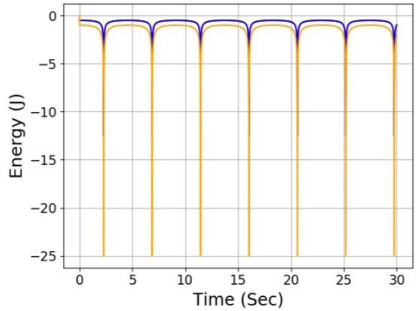
- Integrator Weaknesses:
  - Fixed time step (dt) makes it possible to miss finer levels of physical interaction when particles are near each other
  - “Total Energy” plot is sinusoidal instead of constant
  - Limited to two/three bodies

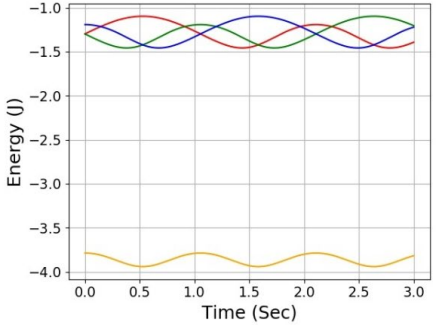
2- And 3- Body Location Plots





2- And 3-Body Energy Plots





Location Plots Key

Each curve (Red, Green, Blue) represents the path/behavior of a different particle.

Energy Plots Key

Red, Green Blue – Total Particle Energies
Yellow – System Total Energy

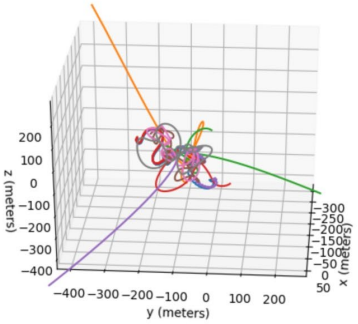
Adaptive Time-Stepping Algorithmic Structure

- To fix the possibility of missing physical interaction with a time step that is too large, we decided to add an adaptive time-stepping algorithm to the original C code. The new algorithm computes three values each loop – the locations of the particle with the original time step, a doubled time step, and a halved time step.
  - If the distance between the locations of the halved time step and the original is too large -> the **halved** time step becomes the new default
    - Improves the accuracy of the integrator during close particle interactions
  - If the distance between the locations of the doubled time step and the original is too small -> the **doubled** time step becomes the new default
    - Helps speed up the computation time of the algorithm

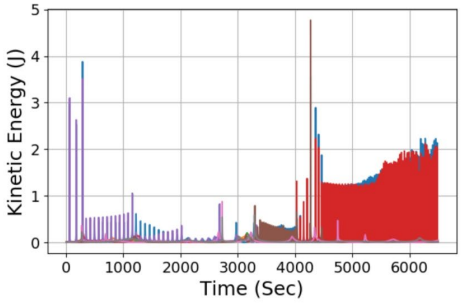
Jumping to N-Bodies/8 Body Test Case

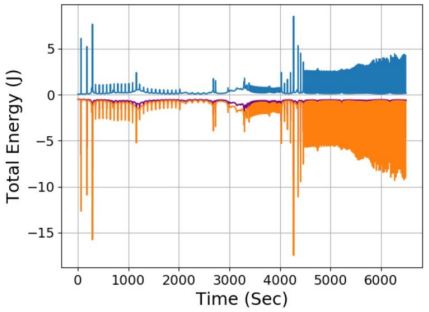
- Next, the leapfrog integrator code was generalized to plot any number of bodies for a set time interval
- Tested with eight bodies distributed randomly within a 100-meter sphere and given a random velocity between -0.1 m/s to 0.1 m/s
- Test Notes:
  - Couple of binary sub-systems and three particle ejections
  - “Combined Energy” plot (purple) is fairly level

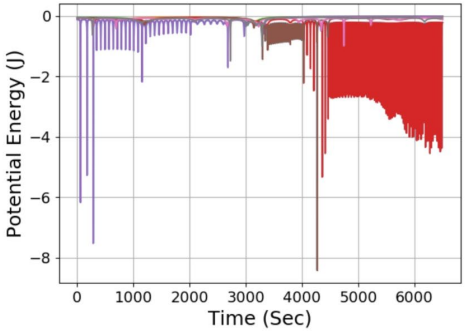
8-Body Location Plot



8-Body Kinetic/Potential Energy Plots







Total Energy Key

Orange/Green – Total Kinetic/Potential Energy

Blue - Combined Energy (Potential + Kinetic)

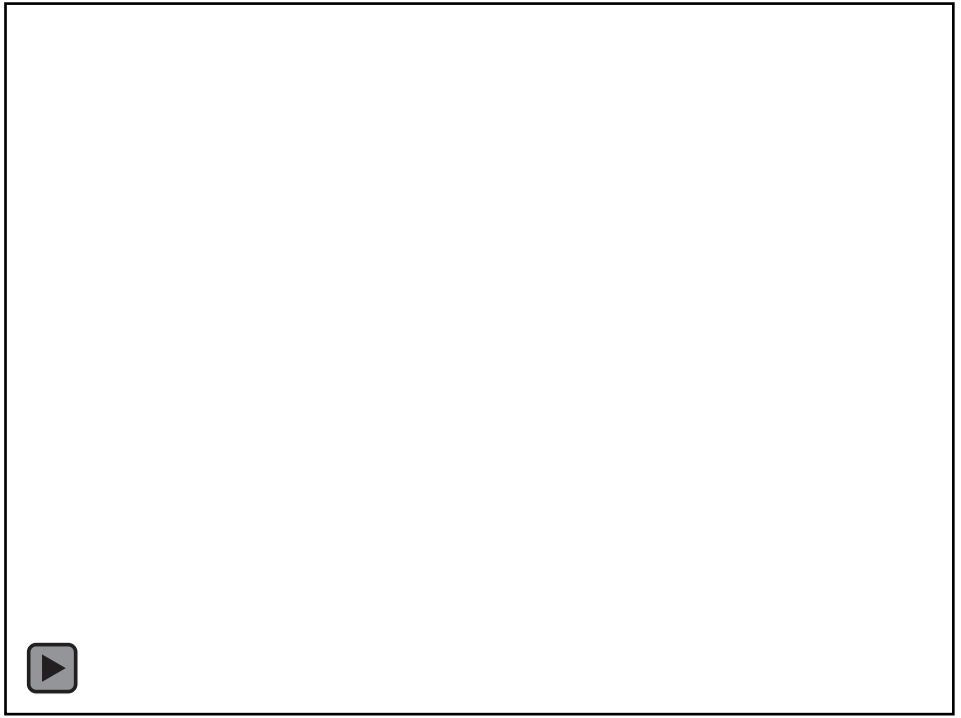
Location, KE, PE Key

Each curve represents the path/behavior of a different particle, with the eight different colors corresponding across the three plots

Final Animation – Integrator Strengths and Weaknesses

- Animated to model five of the particles in the eight-body system (~6.5k seconds, ~330k data points)
- Blue/Purple particle binary gets separated by close interactions (~3.1k sec) and eventually yields a Blue/Red particle binary
- Absence of pseudo-force commonly found in other integrators allows for the modeling of these binary captures/transfers

Animated 8-Body Location Plot (5 Particles)



Conclusion and Future Directions

- Conclusion
  - A user-friendly, adaptive leapfrog integrator that can be used to simulate a wide range of self-gravitating systems consisting of a large number of particles – useful for studying the statistical mechanics of gravitational/electromagnetic systems.

- Future Directions
  - Swap out the Leapfrog scheme with other schemes (ex. Runge-Kutta) and compare the difference in location and energy plot
  - Determine the stability or stochasticity of systems in question – take each particle and add trace “ghost” particles that move under the influence of forces already existing in the system with slightly perturbed initial velocities.

Acknowledgements

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Thank you!